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PROBABILITY OF COLLISION WITH SPECIAL PERTURBATIONS DYNAMICS USING THE MONTE CARLO METHOD

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Special perturbations-based Monte Carlo methods were used to investigate approaches for estimating the probability of collision between two satellites. Sample populations for each satellite are produced via the covariance at epoch, then each sample is propagated to the time span of interest where the closest point of approach between samples is determined. The output of the Monte Carlo analyses produce a cumulative distribution function of miss distance, which, given the combined radius of the two satellites, can be translated into a probability of collision estimate. Comparisons are made against an analytical method and a two-body Monte Carlo method for LEO and GEO orbit cases. Additionally, the impact of covariance realism is considered through the introduction of a scale factor to the covariance matrix. The results indicate that, although the analytic method matches the Monte Carlo methods at the time of closest approach for the LEO case, the probability of collision calculations are in error likely because the propagated Cartesian covariance used as input do not adequately represent the orbit error probability distributions; these errors may be smaller, however, than errors due to covariance realism in general. A high performance computing framework called Collision and Conjunction Analysis (CoCoA), using shared or distributed memory, was utilized to generate results in a timely manner.

INTRODUCTION

The United States relies on space for both defense and civil purposes more than any other nation. Maintaining free access to space is of critical importance for safe operations of satellites (weather monitoring, television, GPS, Earth observation, etc.). As the number of orbiting space objects has grown over the past 50 years, the risk of inadvertent collisions between satellites has also grown. Since the unanticipated collision of an active US Iridium communications satellite with an inactive Russian satellite in February, 2009, there has been heightened awareness of these risks and the potential impacts they have on our ability to maintain free access to space and safe satellite operations.

The ability to accurately predict the probability of collision between any two satellites is one of the most important space situational awareness products for two reasons. First, for maneuverable satellites, it provides the actionable information required on whether a collision avoidance maneuver is warranted. Second, for all cases, it allows for the posturing of space surveillance resources to

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observe and catalog potential debris, which is critical because debris pieces can pose a collision risk to other satellites.

Accurately calculating the probability of collision is not straightforward and requires the integration of a multi-dimensional probability distribution function over an overlap in multi-dimensional space. In practice, various simplifying assumptions are made which allow for analytical estimates to be made.[1–5] These assumptions may not always be appropriate and thus rigorous verification and validation is required.

Using a Monte Carlo approach to this problem has the potential to eliminate several of the deficiencies of simplified analytical methods. In addition to allowing for nonlinear relative motion, using a Cartesian covariance, which has been shown to be a poor choice for orbit error distribution, can be avoided.[6,7] Of course, even with the Monte Carlo approach, the assumption of covariance realism at epoch is inherent. The impact of covariance realism is separately considered in this paper through the introduction of a scale factor to the covariance matrix.

Monte Carlo estimation has been applied to the probability of collision problem previously,[8] however this work differs in two important ways. The study conducted by Alfano uses a basic two-body dynamical model to assess the performance of various analytical methods proposed, while this work utilizes a full fidelity Special Perturbations (SP)-based Monte Carlo method as truth. Additionally, the use of high performance computing makes the SP-based Monte Carlo approach practical as a stand-alone tool if required.

A high performance computing framework called Collision and Conjunction Analysis (CoCoA), using shared or distributed memory, was utilized to generate results in a timely manner. CoCoA provides the ability to perform Monte Carlo-based probability of collision calculations using operationally relevant SP orbit propagators as well as, through slight modification, all-on-all conjunction analysis.

This paper describes the formulation of the SP-based Monte Carlo approach, which is considered truth for the analysis where the samples are taken near the orbit determination observation span. Comparisons are then made to Alfano's analytic approach[8] as well as Monte Carlo approaches utilizing two-body and SP-based propagation near time of closest approach. Finally, the effect of scaling the covariance before sampling is revisited as a measure of the sensitivity of this calculation to errors in the uncertainty distribution.[9]

IMPLEMENTATION

The Monte Carlo analysis samples the orbit error distribution of each satellite at an epoch time and propagates them to find the time and distance of closest approach. Two different approaches are taken: 1) an all-on-all sample comparison implementation well suited to shared memory computation, and 2) an individual sample-on-sample comparison implementation better suited to distributed memory computations.

In both approaches, two satellite states, along with the corresponding covariance matrices, are given. The sample space for each satellite is populated by multiplying the Cholesky decomposition of the covariance matrix by a vector with random Gaussian-distributed elements, then adding this to the nominal state. The gasdev routine from Numerical Recipes was used to generate the random vector of the length of the satellite state.[10] The result is a Gaussian distribution of sample states in position, velocity, and solar radiation pressure and drag coefficients. Tests were performed where the covariance was empirically recovered from the sample states and was shown to match the covariance used to generate the sample states within expected tolerances. It is important to note that the sampling is done in Cartesian space, which may have an impact on some of the results.

The sampling of the initial conditions is done either at some epoch or at the nominal time of closest approach. For the epoch state case, each sample is independently propagated to the nominal time of closest approach (t_{CA}), and a vector table is created for the time-frame $t = t_{CA} \pm P/4$, where P is the period of the nominal satellite. In the analysis considered here the orbit epoch is assumed to be five days prior to t_{CA}.

For the all-on-all approach, all samples for each satellite are propagated and each sample from the first population is then analyzed against every sample from the second, and the minimum relative distance is found. The result of the complete all-on-all analysis is a cumulative distribution function (CDF) of miss distance, which is equivalent to the probability of collision distribution as a function of combined object radius. This is conceptually shown in Figure 1. The advantage to this approach is that $2N$ propagations are required to produce N^2 close approach samples, which is dramatically more efficient than the sample-on-sample approach. The disadvantage of this approach is that not all of the are samples completely independent. To study conjunction analysis with CoCoA, the vector

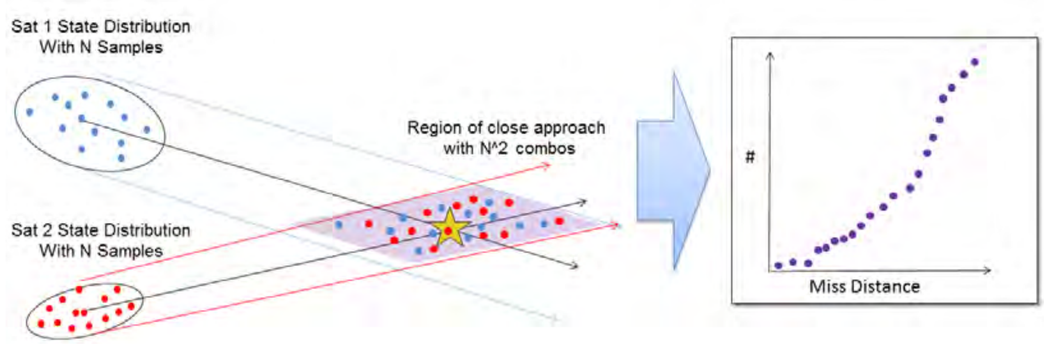


Figure 1. All-on-all Monte Carlo conjunction analysis

table and comparison loops are simply modified to compare trajectories for N satellites as opposed to N trajectory samples for two satellites.

SP-based orbit propagation for the all-on-all approach is achieved using the Naval Research Laboratory's SPeCIAL-K software,[11] originally developed for operational use at DSC2 (Distributed Space Command and Control) - Dahlgren. The SPeCIAL-K software suite uses a Gauss-Jackson special perturbations predictor-corrector as its integrator, and permits a variety of force models and control parameters for orbit determination and propagation. The propagator application, known as EphGen, produces a satellite ephemeris at either uniform or non-uniform time steps, in any one of six available coordinate systems. In its operational configuration, EphGen extracts the epoch state data, force model selections, space environment data, and time constants from an Oracle database. However, the current application requires higher performance than is possible when using a database. Therefore, in order to improve speed and accommodate the large number of runs required, the database queries are replaced with reads from a set of flat files. In addition to the SP approach, an analytic two-body propagator version is also available using software from Vallado.[12]

Because all trajectories are propagated and then stored for comparison, this approach is well suited to shared memory computing environments. The Air Force Research Laboratory Defense Supercomputing Resource Center (AFRL DSRC) Hawk system offers shared memory access with up to 500 processing cores. At the time of this writing, only the all-on-all comparison portion of the code has been parallelized using OpenMP and the SP propagation is performed serially. Efforts are underway to parallelize the propagation portion as well. Because of this current limitation, cases involving 40,000 close approaches, from 200 orbit propagations, were routinely considered.

The sample-on-sample approach avoids the correlation between samples of the all-on-all approach by only comparing one sample from the Satellite 1 population to one sample from the Satellite 2 population. This makes each pair of trajectory samples and the resulting close approach independent of all others. This approach is well suited to distributed memory computing systems but requires $2N$ propagations to produce N close approach samples and thus requires significantly more computations than the all-on-all approach. The SP orbit propagation for the sample-on-sample approach was provided by the Linux R&D version of the Goddard Trajectory Determination System[13] used by the Air Force Maui Optical and Supercomputing site.

Because each pair of propagations and resulting comparison is independent of all others, this approach is easily parallelized on distributed memory computing platforms. Typically, 100,000 close approaches, requiring 200,000 SP orbit propagations, were performed. For this work, the Maui High Performance Computing Center Defense Supercomputing Resource Center (MHPCC DSRC) Mana system was used. Mana offers 9,216 processing cores, although only 40 were used for this study.

Both approaches produced trajectory information for each sample on one minute intervals and saved the position vectors into a memory table. The conjunction analysis algorithm stepped through this vector table minute-by-minute, searching for an instance where the relative position vector between trajectories at time $t-1$ was less than the relative position at time $t-2$ and time t. Once this criterion was met, third order Lagrange interpolation of the trajectories was used along with the Numerical Recipes Brent function to find the time and distance of closest approach;[10] CoCoA actually minimizes the square of the close approach distance. The one minute interval was chosen because analysis showed the interpolation to be accurate to less than 1m. The approach is not elegant, but it is effective.

Analytical Method

The primary assumption in this work is that the SP-based Monte Carlo approach provides the best assessment of the true probability of collision (provided the covariance information is correct, which can be controlled in simulation). Other approaches can then be compared against this truth to test their validity. For this paper, Alfano's method[2] is chosen as a representative analytic approach that can be compared against the Monte Carlo truth. Alfano's method was chosen due to the excellent documentation provided over the years and publications that show it to agree with other analytic approaches.[8]

For Alfano's approach, the satellite states and covariance matrices are propagated to the nominal time of closest approach. Then, the overlap of the two position uncertainty ellipsoids is estimated in two dimensions (defined by the plane normal to the relative velocity vector). Alfano assumes linear relative motion near the time of closest approach and a "fast" conjunction time span (such that the uncertainty can be assumed to be constant). This is computationally inexpensive, and provides reasonably accurate results when the assumptions are met.

In order to compare the output of Alfano's method directly with the method presented here, a cumulative distribution function is generated by varying the combined object radius of the two satellites. This gives a probability of collision for each combined radius, which is essentially identical to a cumulative distribution function of miss distance.

It should be noted that in all cases, Monte Carlo and analytic, the covariance and mean states are referenced to the truth orbits. In practice, one doesn't know truth and the covariance is linearized about the best estimate, represented by the individual samples in this study. One would expect small differences if any of the approaches were referenced to a sample trajectory rather than truth.

Test Cases

Two cases were developed for testing. One involves two coplanar satellites at GEO, while the other involves two LEO satellites with perpendicular orbit planes – one equatorial and one polar. Realistic covariance matrices were generated from a GTDS differential correction process on simulated observational data. The GEO case was meant to reflect a collision between a station-kept GEO satellite and one drifting through the belt. Figure 2 plots the position differences between the satellites for the five days prior to the close approach for the GEO case. Earth-centered, inertial state vectors at the time of closest approach are shown in Tables 1 and 2.

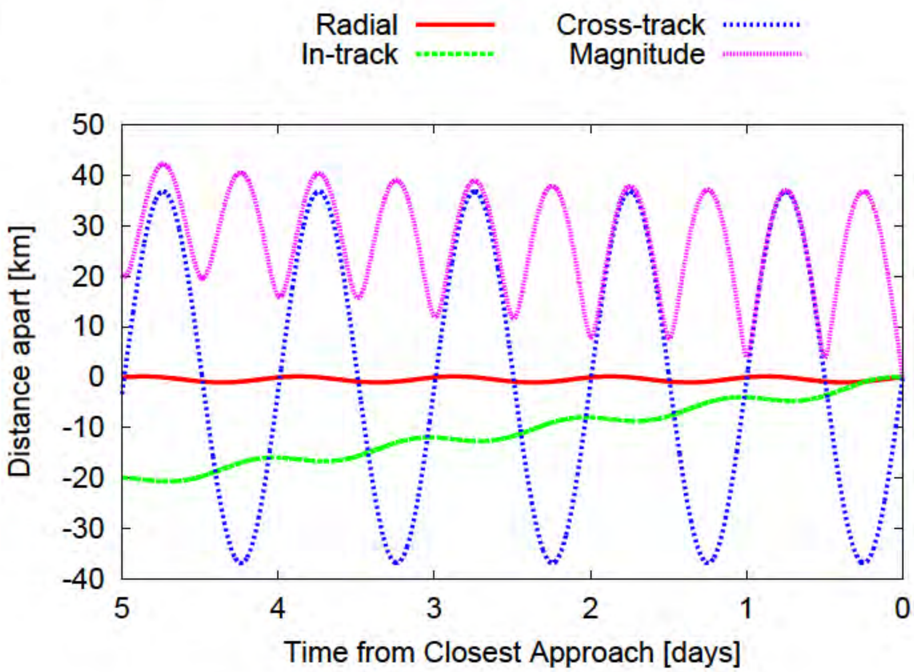


Figure 2. Position Differences for the GEO Case

Efforts were made to establish uncertainties similar to those experienced with SP-based surveillance products. For the LEO case, semimajor axis uncertainty was around 1m and the eccentricity uncertainty was around 0.00001. Out of plane uncertainty is on the order of tens of meters. For the GEO case, the semimajor axis uncertainty was around 10m and the eccentricity uncertainty was around 0.0002. Out of plane uncertainty is on the order of hundreds of meters. The covariance matrices for the test cases are in the Appendix.

Table 1. LEO test case state vectors at close approach (position in km, velocity in km/s).

	Satellite 1	Satellite 2
X	6999.930	6999.930
Y	0	0
Z	0	0
$\dot{X}$	0	0
$\dot{Y}$	7.546129	0
$\dot{Z}$	0	7.546129
C_d	2.0	2.0
C_r	1.2	1.2

Table 2. GEO test case state vectors at close approach (position in km, velocity in km/s).

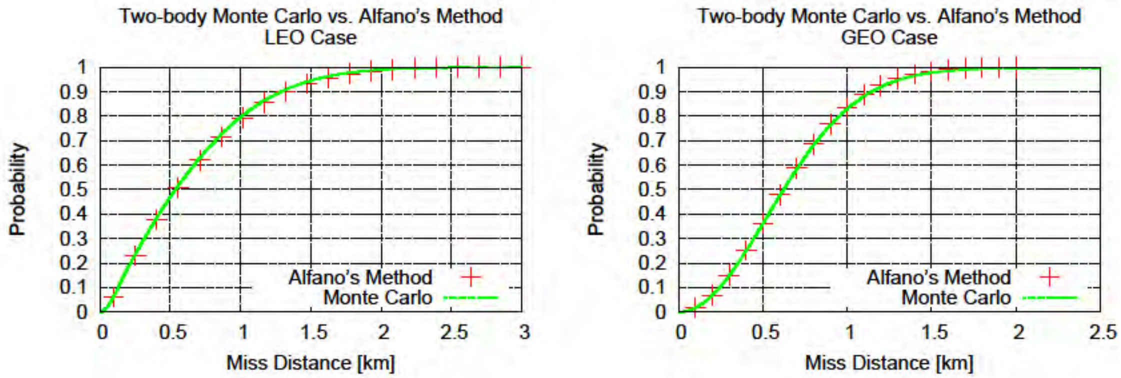
	Satellite 1	Satellite 2
X	-12327.43	-12327.43
Y	40321.23	40321.23
Z	0	0
$\dot{X}$	-2.940347	-2.940340
$\dot{Y}$	-0.8989545	-0.8989202
$\dot{Z}$	0	0.0026832
C_d	N/A	N/A
C_r	1.2	1.2

For the LEO cases, a 40×40 WGS-84 gravity field and Jacchia-70 drag model with F10.7cm = 150 and Ap = 10 were used, as well as luni-solar gravity and solar radiation pressure. The drag coefficient was included in the state but the SRP was assumed to be perfectly known. The SRP coefficient is included in the covariance matrices in the Appendix but one can see that the correlation terms are zero and no variations to the SRP were included in the LEO Monte Carlo analysis. Similarly, for the GEO cases, an 8×8 WGS-84 gravity field, luni-solar gravity, and solar radiation pressure was used. Atmospheric drag was not used in the GEO cases; the drag coefficient is included in the covariance matrices in the Appendix but one can see that the correlation terms are zero and no drag accelerations were included in the GEO Monte Carlo analysis. In all cases, the satellites were assumed to be spherical with an area to mass ratio of 0.01.

RESULTS

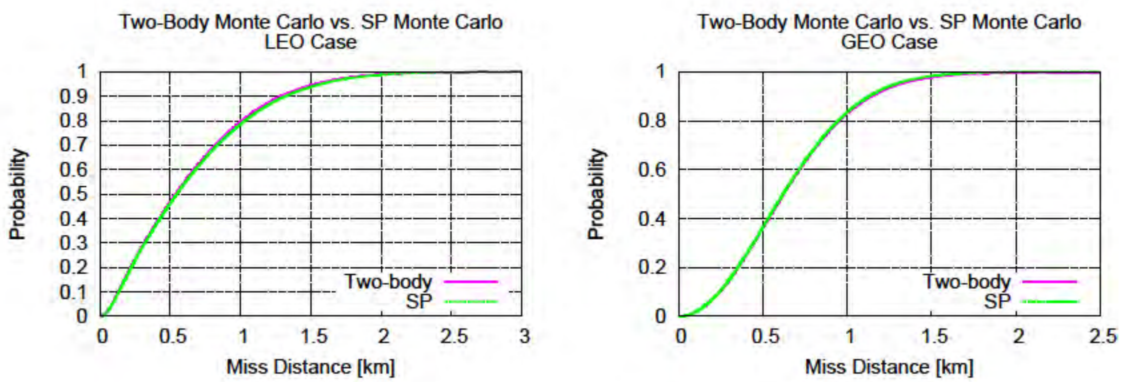
An initial comparison is made between Alfano's analytical method and a two-body version of the Monte Carlo method, with sampling done at t_{CA}. By using two-body dynamics, and not propagating to t_{CA} from some epoch time, this provides a case which is suitably similar to Alfano's method to use for verification and validation. As expected, Figures 3(a) and 3(b) show good agreement between the two approaches. Additionally, changing the seed for the random number process results in overlapping Monte Carlo distributions – evidence that the results are statistically consistent with a satisfactory number of samples for these purposes.

Comparisons are then made between the two-body and special perturbations propagators. Again, both objects are sampled at t_{CA}. As a result, there is only a quarter revolution of propagation time for perturbing forces to manifest themselves, and good agreement is to be expected. This behavior is shown in Figures 4(a) and 4(b).



(a) LEO case. (b) GEO case.

Figure 3. Two-body Monte Carlo comparison to Alfano's analytical method.



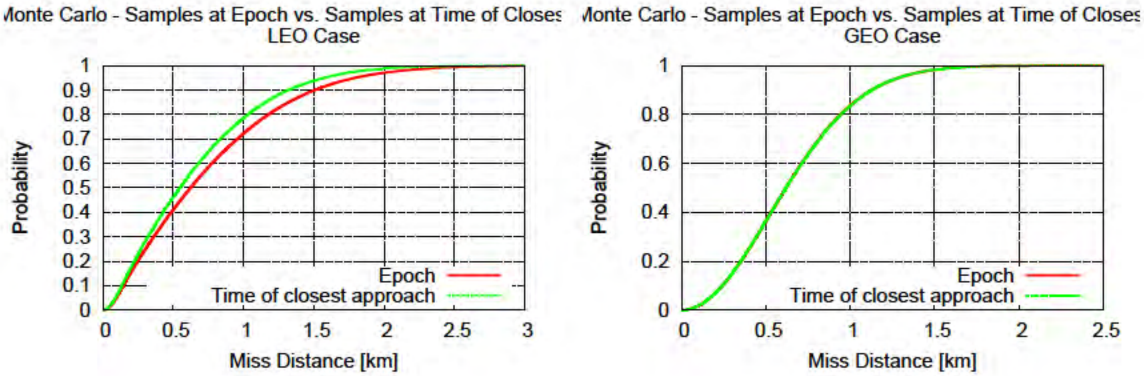
(a) LEO case. (b) GEO case.

Figure 4. Comparison between two-body and special perturbations propagators.

Next the question of covariance realism is tested by comparing Monte Carlo runs of samples generated at epoch ($t_{CA} - 5$ days) against those generated at t_{CA} from the propagated covariance matrix. By propagating the sample "cloud" from epoch, the covariance near t_{CA} may be measured by proxy in the state distribution of the samples.

For the GEO case, shown in Figure 5(b), there is excellent agreement. However for the LEO case, shown in Figure 5(a), there is a small but significant difference when the states are sampled at epoch. It should be noted that, with these results, the CoCoA software was thoroughly scrubbed and cases were re-run without drag and the results stood. The differences are likely due to the use of a Cartesian covariance matrix. Previous work has shown that the Cartesian basis is a poor choice for representing orbit error distributions and even with a relatively small uncertainty, nonlinearity results in the Cartesian covariance no longer representing the true error distribution within a relatively small number of orbits.[7] This could probably be avoided if the covariance were transformed into element space and the samples taken there. This was not completed in time for the publication of this paper.

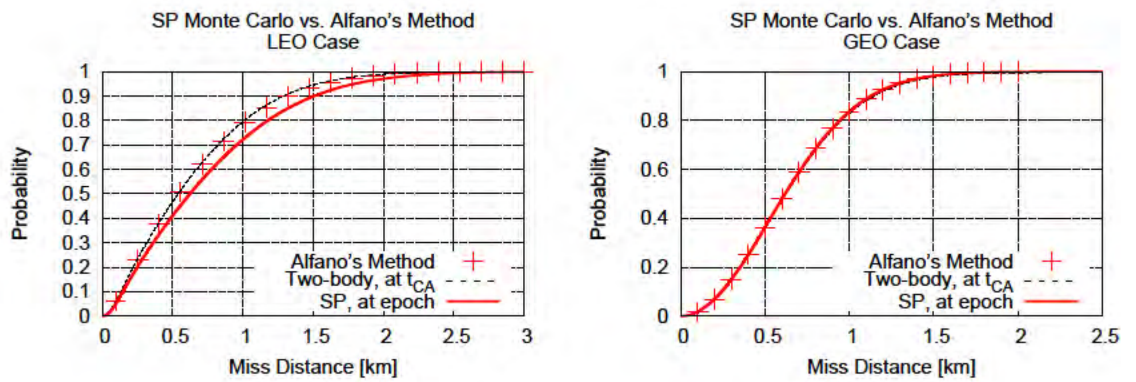
Figures 6(a) and 6(b) show a direct comparison between the SP Monte Carlo method and Al-



(a) LEO case.

(b) GEO case.

Figure 5. Difference between samples taken at epoch and samples taken at t_{CA}.



(a) LEO case.

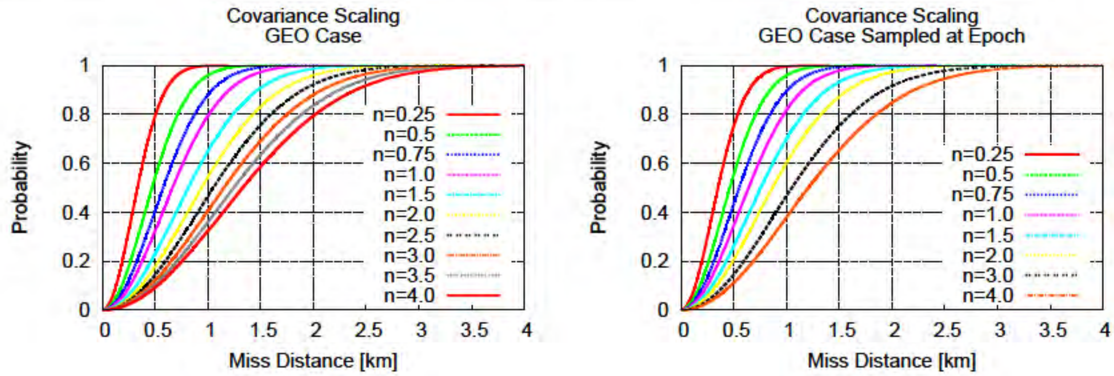
(b) GEO case.

Figure 6. Difference between samples taken at epoch and samples taken at t_{CA}.

fano's analytical method. Although perturbing forces have sufficient time in which to manifest themselves in the motion of the satellites, relatively close agreement is observed for the GEO case. This indicates that the assumptions in the analytic approach are valid for this test case. For the LEO case, there are noticeable differences, again likely due to the Cartesian position uncertainty representation. Differences on the order of 20% can be observed between all the approaches that use the Cartesian uncertainty at t_{CA} and the $t_{CA} - 5$ days truth SP-based Monte Carlo results.

Covariance Scaling

In this section, the covariance matrix of each satellite is uniformly scaled by some multiple n. As a result of the nominal miss distance for the cases being close to zero, scaled covariance has the expected effect: a lower covariance implies lower state uncertainty, meaning a greater chance of collision. For the cases with nonzero nominal miss distance, the expected result is that the estimated probability of collision peaks at some $n > 1$ before settling back down to some lower estimate as the uncertainty grows higher. Figure 7(a) illustrates the effect of a scaled covariance on the coplanar GEO case at t_{CA}. As expected, the probability of collision is lower when $n < 1$ and higher when $n > 1$. A lower covariance corresponds with lower uncertainty; in a case in which the unperturbed states will collide, lower state uncertainty results in a higher probability distribution. Similar behavior is shown in Figure 7(b) for the GEO case sampled at epoch.



(a) Sampled at t_{CA}.

(b) Sampled at t_0.

Figure 7. GEO case, with covariance matrix scaled by n.

One can see in these figures that if the covariance is in error by a factor of 2-4, which can be the case for orbit determination with poorly modeled sensor noise characteristics and unmodeled or poorly modeled process noise, the probability of collision changes significantly. Previously published work has shown several orders of magnitude error in probability of collision with similar level covariance scale errors.[5] Thus, while the comparisons in this work have shown differences between the assumed truth and analytic probability of collision approaches, these differences are likely to be small when considered in the relative to the impact of covariance inaccuracy.

Discussion of Approaches

As discussed in the implementation section, the all-on-all approach stores trajectory information in shared memory. This dramatically reduces the number of trajectories that need to be propagated in order to get a desired number of close approach samples. Runs were made using both approaches where the number of samples was increased from 10,000 to 100,000 or 1,000,000. The result found that coarse probability of collision CDF behavior was observed at 10,000 samples but 100,000 was required for finer consistency, particularly at the lower tail of the distribution. It was interesting to note that this held for both approaches. Given a modern, shared memory system of 500 cores, it is expected that 100,000 conjunction sample (2000 total propagations) would only require a couple seconds even with the SP dynamics and unrefined search approach used here.

CONCLUSIONS

The method of SP-based Monte Carlo analysis to determine probability of collision between two satellites was introduced and analyzed. Given the covariance generated by a differential correction at an epoch near the simulated observation span, random samples can be generated for each satellite of interest and propagated to the point of closest approach. The cumulative distribution function of these close approaches forms the basis of truth to compare probability of collision calculations.

Alfano's analytic approach, which is representative of its class, along with other Monte Carlo implementations epoched near the time of closest approach were compared against this truth for a LEO and GEO collision case.

All probability of collision methods considered require an accurate representation of the orbit error distribution at some point in time. It is believed that the Cartesian representation of the covariance near the time of closest approach for the LEO test case did not adequately reflect the orbit error distribution and thus the methods that used this covariance did not agree with the SP-based Monte Carlo derived from the differential correction epoch. These differences might be avoided if the covariance in element space were used. For the GEO case, where the time of closest approach and differential correction epoch are only five orbits apart, all methods are in agreement.

Additional analysis was performed to examine the impact of covariance errors on the probability of collision. As in previous work, significant probability of collision differences were observed with reasonable covariance scale errors. These differences are larger than the differences observed between the analytic probability of collision method and the assumed truth SP-based Monte Carlo results. Thus, for practical purposes, limitations in the analytic approaches for probability of collision calculations are likely acceptable.

If one wanted to implement the Monte Carlo method in place of an analytic approach, the results show that a 2-body Monte Carlo provides similar results to the SP-based Monte Carlo when initiated near the time of closest approach; thus, the Monte Carlo approach can be computationally cheap if sampling the covariance in element space provides the expected results. Even if that were not the case, through the use of the all-on-all trajectory sample approach and the use of shared memory high performance computing, the full SP-based Monte Carlo approach using the covariance at the differential correction epoch is still viable.

Future work in this area remains rich. First, this work needs to be extended to use an element-based covariance sampling approach. Next, it is suggested that a 100,000 by 100,000 conjunction analysis, already demonstrated with CoCoA, be used to generate test cases for this type of analysis. The result would be a broad spectrum of test scenarios where the approaches could be better vetted. Even though the GEO test case here showed agreement between methods, Alfano has shown similar cases where the analytic method was not as accurate.[8] Additionally, the Monte Carlo analysis for truth could occur in the observation simulation phase, thus capturing the effects of poorly modeled sensor errors, mismodeled dynamics, and the impact of linearizing about the estimated trajectory rather than truth.

It is also interesting to note that the two-body Monte Carlo results match the SP-based Monte Carlo when initiated at the time of closest approach. If the covariance were sampled in element space and matched the distribution produced when sampling at epoch, this would imply that a 2-body-based Monte Carlo may be sufficient and would dramatically reduce computational requirements when combined with the all-on-all approach.

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Table 3. Inertial state (X1 & X2) and covariance (P1 & P2) for the LEO case at epoch (units in m and s).

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	C_d	C_r
X1	-2886471.849	-6369105.601	-1622.975846	6876.478296	-3127.069168	-1.526169927	2.0	1.2
P1	5727.82256925319	-1860.43610352951	-61.8099547047470	3.35461158165259	5.53528911401309	0.0396735787177515	0.773795597646805	0.0
	-1860.43610352951	921.758917911844	39.3993696914285	-0.831421297110148	-1.98906916718038	-0.0314969778679955	-0.318751909296129	0.0
	-61.8099547047470	39.3993696914285	711.599423912145	-0.0145505863881775	-0.0574964470851198	0.161998382970665	-0.00373407964795500	0.0
	3.35461158165259	-0.831421297110148	-0.0145505863881775	0.00222206517141115	0.00318721285353524	7.33660241038197e-06	0.000387576021836960	0.0
	5.53528911401309	-1.98906916718038	-0.0574964470851198	0.00318721285353524	0.00567910465147520	4.55328569832028e-05	0.000736837270184263	0.0
	0.0396735787177515	-0.0314969778679955	0.161998382970665	7.33660241038197e-06	4.55328569832028e-05	0.000798745981278186	6.47178344127764e-06	0.0
	0.773795597646805	-0.318751909296129	-0.00373407964795500	0.000387576021836960	0.000736837270184263	6.47178344127764e-06	0.00995715032805239	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0
X2	3447224.77200000	-109.845795100000	-6085547.17400000	6566.20862900000	-0.283856156100000	3715.93578400000	2.0	1.2
P2	101016.481583947	110.411619490341	41885.8621282953	-56.6415811560851	0.219989726228513	71.1499507140171	-2.43473629559525	0.0
	110.411619490341	61.4465996244871	46.1888349105083	-0.0619800677619381	0.116501050601869	0.0786695447439041	-0.00388683986917559	0.0
	41885.8621282953	46.1888349105083	17376.8046273387	-23.4813915020292	0.0920241904797824	29.5112070688341	-0.957182176632926	0.0
	-56.6415811560851	-0.0619800677619381	-23.4813915020292	0.0317646469748266	-0.000123521154148010	-0.0398946438144661	0.00134456091786580	0.0
	0.219989726228513	0.116501050601869	0.0920241904797824	-0.000123521154148010	0.000221320637294872	0.000156799594536602	-6.46753142629845e-06	0.0
	71.1499507140171	0.0786695447439041	29.5112070688341	-0.0398946438144661	0.000156799594536602	0.0501319731173491	-0.00155380710920426	0.0
	-2.43473629559525	-0.00388683986917559	-0.957182176632926	0.00134456091786580	-6.46753142629845e-06	-0.00155380710920426	0.00830078581283432	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

Table 4. Inertial state (X1 & X2) and covariance (P1 & P2) for the LEO case at t_{CA} (units in m and s).

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	C_d	C_r
X1	6999930.0	-0.000396011212700000	-8.83983045600000e-08	4.27856222700000e-07	7546.12930200000	3.35685057600000e-10	2.0	1.2
P1	267.430433963736	3306.90152722423	22.4224553212989	-3.65874188418330	-0.289289333775819	0.00292029792486864	-0.0551912532369451	0.0
	3306.90152722423	797447.999221821	-1038.71553004111	-865.468574018520	-4.45605767875449	0.293619707431429	73.5931065350292	0.0
	22.4224553212989	-1038.71553004111	577.800726730687	1.11219091252451	-0.0240334005011102	0.0962493445222890	0.00476436423316785	0.0
	-3.65874188418330	-865.468574018520	1.11219091252451	0.939654191322558	0.00490756620290308	-0.000310952730762856	-0.0794031513896599	0.0
	-0.289289333775819	-4.45605767875449	-0.0240334005011102	0.00490756620290308	0.000314144538929887	-3.14357165417681e-06	-4.88696579775607e-05	0.0
	0.00292029792486864	0.293619707431429	0.0962493445222890	-0.000310952730762856	-3.14357165417681e-06	0.000954164174876119	-5.38480413771032e-06	0.0
	-0.0551912532369451	73.5931065350292	0.00476436423316785	-0.0794031513896599	-4.88696579775607e-05	-5.38480413771032e-06	0.00995715040022531	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0
X2	6999930.0	1.77017309200000e-05	-0.00582648686700000	6.29098023700000e-06	-2.13354590200000e-09	7546.12930200000	2.0	1.2
P2	7429.60536947151	62.47372119200800	4669.07128321961	-7.38920210856336	-0.000741681619327706	-8.00938292274633	-0.924719611684867	0.0
	62.47372119200800	252.214260056835	31.0375751443411	-0.0531524414298439	-0.000438355627032347	-0.0673057979278591	-0.0109186377474832	0.0
	4669.07128321961	31.0375751443411	489605.213375715	-530.737978410173	0.0517402539381228	-5.84336878485163	62.0688321721617	0.0
	-7.38920210856336	-0.0531524414298439	-530.737978410173	0.576068713820366	-5.57133870296802e-05	0.00884143587409215	-0.0668123004447391	0.0
	-0.000741681619327706	-0.000438355627032347	0.0517402539381228	-5.57133870296802e-05	1.13220782021938e-07	7.09738987614260e-07	7.21629234807471e-06	0.0
	-8.00938292274633	-0.0673057979278591	-5.84336878485163	0.00884143587409215	7.09738987614260e-07	0.00863575875491351	0.000891669606007147	0.0
	-0.924719611684867	-0.0109186377474832	62.0688321721617	-0.0668123004447391	7.21629234807471e-06	0.000891669606007147	0.00831362081031177	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

Table 5. Inertial state (X1 & X2) and covariance (P1 & P2) for the GEO case at epoch (units in m and s).

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	C_d	C_r
X1	-8751615.54000000	41247456.3100000	-1305.11495900000	-3007.56724800000	-638.323990100000	-0.517978136000000	2.0	1.2
P1	66029.0073164945	-73369.5882564336	11.6993163826219	-8.65258455021962	10.9807344044836	-0.00163412894707246	0.0	-0.110304850132476
	-73369.5882564336	173890.744451466	-20.9954223708698	18.2119236135621	-20.7460520125912	0.00400992601955818	0.0	0.501019164293334
	11.6993163826219	-20.9954223708698	18620.9652529696	-0.00217243228877826	0.00291340587883324	-1.17306413710260	0.0	0.00168249946306216
	-8.65258455021962	18.2119236135621	-0.00217243228877826	0.00194453029752869	-0.00228554418607030	3.81936490149029e-07	0.0	6.01430425364982e-05
	10.9807344044836	-20.7460520125912	0.00291340587883324	-0.00228554418607030	0.00287022848578103	-5.01576533556979e-07	0.0	-0.000159777226491056
	-0.00163412894707246	0.00400992601955818	-1.17306413710260	3.81936490149029e-07	-5.01576533556979e-07	0.000275971623000373	0.0	-1.36648132835624e-06
	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
	-0.110304850132476	0.501019164293334	0.00168249946306216	6.01430425364982e-05	-0.000159777226491056	-1.36648132835624e-06	0.0	0.00999999933215962
X2	-8732150.62100000	41251544.1100000	-4605.08531900000	-3007.86176000000	-636.869562200000	2.15433623100000	2.0	1.2
P2	65941.8090322062	-73292.3740702832	-9.34745090777441	-8.63512529801082	10.9681425924250	0.00231666060984573	0.0	-0.110045017870909
	-73292.3740702832	173928.259289340	1.39761248239079	18.2008400797119	-20.7475165796793	-0.00758470808150835	0.0	0.501170892708560
	-9.34745090777441	1.39761248239079	18624.7531384178	-0.000496127287787739	-0.00137345655046966	-1.17328283806881	0.0	0.00165062678258979
	-8.63512529801082	18.2008400797119	-0.000496127287787739	0.00194158759542778	-0.00228353893284958	-5.76414048612804e-07	0.0	6.00382118628679e-05
	10.9681425924250	-20.7475165796793	-0.00137345655046966	-0.00228353893284958	0.00286976360447090	9.35313851784730e-07	0.0	-0.000159644677868321
	0.00231666060984573	-0.00758470808150835	-1.17328283806881	-5.76414048612804e-07	9.35313851784730e-07	0.000275933105226532	0.0	-1.37899937420720e-06
	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
	-0.110045017870909	0.501170892708560	0.00165062678258979	6.00382118628679e-05	-0.000159644677868321	-1.37899937420720e-06	0.0	0.00999999933162021

Table 6. Inertial state (X1 & X2) and covariance (P1 & P2) for the GEO case at t_{CA} (units in m and s).

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	C_d	C_r
X1	-12327437.2800000	40321230.5200000	-1.08868535900000e-08	-2940.34760900000	-898.954477800000	-5.04558958100000e-11	2.0	1.2
P1	128219.957343369	-2049.58491464374	5.80693787037835	-6.26764645575325	10.2838524676183	-0.000943401429861959	0.0	-0.152938473871233
	-2049.58491464374	149094.797418491	-5.70285406535000	15.0358806244609	-13.6310079367302	0.000685513969316458	0.0	0.467679692321669
	5.80693787037835	-5.70285406535000	16025.1544162400	-0.000859546392185269	0.00103778616738773	-0.938102380992324	0.0	-0.000463134292719960
	-6.26764645575325	15.0358806244609	-0.000859546392185269	0.00183197367475363	-0.00194564672766365	1.29672029323356e-07	0.0	-8.43136634217581e-06
	10.2838524676183	-13.6310079367302	0.00103778616738773	-0.00194564672766365	0.00234317608447650	-1.93444335650878e-07	0.0	0.000110029312907339
	-0.000943401429861959	0.000685513969316458	-0.938102380992324	1.29672029323356e-07	-1.93444335650878e-07	0.000289720309522038	0.0	-1.18630797561620e-06
	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
	-0.152938473871233	0.467679692321669	-0.000463134292719960	-8.43136634217581e-06	0.000110029312907339	-1.18630797561620e-06	0.0	0.00999999933249158
X2	-12327437.2800000	40321230.5100000	4.65229813900000e-07	-2940.34077700000	-898.920237800000	2.68316577400000	2.0	1.2
P2	128210.646029239	-1998.48561455208	-87.1831679924693	-6.26260343601102	10.2804305150453	0.000926639167601709	0.0	-0.152366124786121
	-1998.48561455208	148821.401215599	-38.6102226708598	15.0099307515128	-13.6150129294012	-0.00991400788589375	0.0	0.465462673913356
	-87.1831679924693	-38.6102226708598	16016.2517980983	-0.000322271991091716	-0.00422768911067702	-0.937064282797995	0.0	-0.000484455732192042
	-6.26260343601102	15.0099307515128	-0.000322271991091716	0.00182957805732893	-0.00194437775950010	-8.03667361723331e-07	0.0	-8.63447428734383e-06
	10.2804305150453	-13.6150129294012	-0.00422768911067702	-0.00194437775950010	0.00234311972553782	1.06295171597836e-06	0.0	0.000110125179925655
	0.000926639167601709	-0.00991400788589375	-0.937064282797995	-8.03667361723331e-07	1.06295171597836e-06	0.000289750060314243	0.0	-1.20822219202136e-06
	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
	-0.152366124786121	0.465462673913356	-0.000484455732192042	-8.63447428734383e-06	0.000110125179925655	-1.20822219202136e-06	0.0	0.00999999933129540